a simple procedure for detecting periodically collapsing rational bubbles[*]

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Abstract

This paper proposes a new procedure for detecting the presence of periodically collapsing rational bubbles through an analysis of the properties of the relevant observable time series. The procedure is based on random-coefficient autoregressive models. An empirical application of the procedure to German hyperinflation data is examined and discussed.

Keywords: Rational bubbles; Stochastic unit root; Time-varying coefficients.

JEL Classification: C22, E31, G12.

1 Introduction

It is by now well documented that tests for unit roots and cointegration may fail to detect the presence of explosive rational bubbles that collapse periodically. In an important paper, Evans (1991) highlighted the problem by demonstrating that standard unit-root and cointegration tests for asset prices and underlying fundamentals can erroneously lead to acceptance of the no-bubble hypothesis when prices contain an explosive stochastic bubble which collapses from time to time. The essence of the problem lies with the fact that collapsing bubbles only exhibit characteristic explosive bubble behaviour during their expansion phase and hence unit-root and cointegration tests are powerful enough to detect the bubble only when its expansion phase lasts for most of the sample period under investigation. Since, therefore, the problem is essentially one of identifying the expanding periods from the collapsing ones, Hall et al. (1999) proposed using a unit-root test based on an autoregressive model with Markov switching parameters (see also Funke et al., 1994).[1] Such a test was shown to have considerable power to detect the presence of periodically collapsing rational bubbles in asset prices (see also van Norden and Vigfusson, 1998). The difficulty, however, is that the test procedure is very computer-intensive since simulation methods need to be used to obtain critical values for the test.

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[1]Alternative test procedures also based on regime-switching regressions were examined by van Norden (1996) and van Norden and Schaller (1993).

This paper proposes an alternative simpler way of detecting the presence of explosive rational bubbles that collapse periodically. Our procedure is based on the class of nonstationary varying-coefficient autoregressive models with a stochastic unit root examined by McCabe and Tremayne (1995), Leybourne et al. (1996), and Granger and Swanson (1997). The basic idea is to first test whether the time series of interest can be described as having a random root with unit mean and then to identify periods during which explosive behaviour consistent with the presence of a bubble is displayed. We illustrate the new method by investigating the possibility of a bubble being present in German hyperinflation data.

2 Testing for a Random Unit Root

The usual indirect way of testing for the presence of explosive rational bubbles is based on checking the order of integration of a given pair of variables. Specifically, if prices (say stock prices or inflation) are not more explosive than the relevant driving fundamental variable (say dividends or money expansion), then it can be concluded that rational bubbles are not present since, if they were, they would generate an explosive component in the respective prices (see, e.g., Diba and Grossman, 1988). Such hypotheses have been typically tested using unit-root tests based on fixed-coefficient autoregressive models for prices and fundamentals. However, in the presence of rational bubbles that collapse periodically (e.g., Evans, 1991) or bubbles that have a stochastic explosive root (e.g., Charemza and Deadman, 1995), models that allow for random coefficient variation are likely to provide a better representation of the dynamics of a series with a bubble component. A model with an autoregressive root that varies randomly around a unit mean yields time series which can occasionally exhibit explosive behaviour and which cannot be made stationary by differencing, behaviour which is characteristic of an explosive bubble.

As a first step, therefore, in detecting the presence of a bubble we propose investigating whether the time series of interest can be described as having a stochastic unit autoregressive root. The test of Leybourne et al. (1996) offers a convenient way of distinguishing between a process with a fixed unit autoregressive root (i.e., an integrated of order one, or $I(1)$, process) and a process with an autoregressive root that exhibits random variation around unity (which we shall refer to as an $I_t(1)$ process). For a time series $\{X_t\}$, the test is based on the random-coefficient autoregressive model

$$X_t - \mu_t - \sum_{i=1}^{p} \phi_i X_{t-i} = \rho_t \left(X_{t-1} - \mu_{t-1} - \sum_{i=1}^{p} \phi_i X_{t-i-1} \right) + \varepsilon_t; \qquad t = 1, \ldots, T;$$

where $\mu_t = \delta_0 + \delta_1 t + \delta_2[t(t+1)/2]$. It is assumed that $\{\rho_t\} \sim \text{i.i.d.}\ (1, \omega^2)$, $\{\varepsilon_t\} \sim \text{i.i.d.}\ (0, \sigma_\varepsilon^2)$ independent of ρ_t, and all the zeros of the polynomial $z^p - \phi_1 z^{p-1} - \cdots - \phi_p$ appear inside the unit circle. In this model, testing $H_0 : \omega^2 = 0$ against $H_1 : \omega^2 > 0$ is equivalent to testing the hypothesis that $\{X_t\}$ is $I(1)$ against the alternative that $\{X_t\}$ is $I_t(1)$.

Assuming that the conditional distribution of X_t given information (in ρ_t and ε_t) available at time $(t - 1)$ is Gaussian, Leybourne et al. (1996) show that the score test for $H_0 : \omega^2 = 0$ against $H_1 : \omega^2 > 0$ rejects for large values of the statistic

$$\hat{Z}_T = T^{-3/2} \hat{\kappa}^{-1} \hat{\sigma}_\varepsilon^{-2} \sum_{t=p+3}^{T} \left(\sum_{j=p+2}^{t-1} \hat{\varepsilon}_j \right)^2 (\hat{\varepsilon}_t^2 - \hat{\sigma}_\varepsilon^2);$$

where $\{\hat{\varepsilon}_t\}$ are the residuals from the least-squares regression of $\Delta X_t = X_t - X_{t-1}$ on $\Delta X_{t-1}, \ldots, \Delta X_{t-p}$, a constant and a linear time trend, $\hat{\omega}^2 = T^{-1}\sum_{t=p+2}^{T}(\hat{\varepsilon}_t^2 - \hat{\sigma}_\varepsilon^2)^2$, and $\hat{\sigma}_\varepsilon^2 = T^{-1}\sum_{t=p+2}^{T}\hat{\varepsilon}_t^2$. Under $H_0 : \omega^2 = 0$, $\hat{\Xi}_T$ has a non-standard asymptotic distribution which depends on $\psi = \mathrm{corr}(\varepsilon_t, \varepsilon_t^2)$. Although this limits the usefulness of the test to situations where ψ is known or ε_t is symmetrically distributed (so that $\psi = 0$), Leybourne et al. (1996, p. 439) note that a test with critical region determined under the assumption that $\psi = 0$ was found to perform well in simulation experiments where $|\psi| \leq 0.7$.

In any case, $\hat{\Xi}_T$ may be modified so that its asymptotic null distribution does not depend on ψ. Such a modified statistic is given by

$$\hat{\Xi}_T^* = (1 - \hat{\psi}^2)^{-1/2}\hat{\Xi}_T - \hat{\psi}(1 - \hat{\psi}^2)^{-3/2}T^{-3/2}\hat{\sigma}_\varepsilon^{-3}\sum_{t=p+3}^{T}\left(\sum_{j=p+2}^{t-1}\hat{\varepsilon}_j\right)^2\hat{\varepsilon}_t;$$

where $\hat{\psi} = [\sum_{t=p+2}^{T}\hat{\varepsilon}_t^2\sum_{t=p+2}^{T}(\hat{\varepsilon}_t^2 - \hat{\sigma}_\varepsilon^2)^2]^{-1/2}\sum_{t=p+2}^{T}\hat{\varepsilon}_t(\hat{\varepsilon}_t^2 - \hat{\sigma}_\varepsilon^2)$. Critical values for a test based on $\hat{\Xi}_T^*$ or $\hat{\Xi}_T$ (with $\psi = 0$) are tabulated in Leybourne et al. (1996, Table 1).

As mentioned earlier, since the existence of a periodically collapsing explosive bubble implies the presence of an explosive autoregressive root during the expansion phase of the bubble, our strategy is to first test the $I(1)$ versus the $I_t(1)$ hypothesis for prices and fundamentals.[2] If the hypothesis of a fixed unit root is rejected, we then estimate random-coefficient autoregressive models for the two series and examine the time-paths of (the sum of) autoregressive coefficients. The presence of an explosive rational bubble is consistent with prices exhibiting explosive behaviour in periods during which fundamentals do not behave in an explosive fashion.

3 The German Hyperinflation

As a practical illustration of the idea discussed before, we investigate the possibility of a rational bubble in monthly German hyperinflation data. Figure 1 plots the natural logarithm of the wholesale price index and money supply (notes in circulation) for the period 1918:12–1924:12.

Table 1 reports the values of the $\hat{\Xi}_T$ and $\hat{\Xi}_T^*$ statistics for the two series. Since, in small samples, the outcomes of the test can be significantly affected by underfitting or overfitting (cf. Leybourne et al. 1996), we present results for $p = 0, 1, \ldots, 5, 6, 12$. Underlined entries are the values of the statistics when p is chosen by means of the familiar Akaike information criterion (AIC) for the least-squares regression of ΔX_t on $\Delta X_{t-1}, \ldots, \Delta X_{t-p}$, a constant and a linear time trend. The results clearly show that the $I(1)$ hypothesis can be rejected in favour of the $I_t(1)$ alternative for both series and for almost all values of p. The $\hat{\Xi}_T$ statistic does not lead to a rejection of the fixed unit root hypothesis when $p = 0$ or $p = 12$ (in the case of prices), but this statistic is arguably the less trustworthy of the two given the relatively large estimated values for ψ in the cases in question.

[2] In a companion paper (Psaradakis et al., 2000), we demonstrate by means of simulation experiments that tests based on $\hat{\Xi}_T$ and $\hat{\Xi}_T^*$ have considerably more power than standard unit-root tests to reject the $I(1)$ hypothesis for prices when the latter have a periodically collapsing bubble component.

Having established that the dynamics of both prices and money supply are characterized by the presence of an autoregressive root that varies randomly around unity, we now proceed to fit a random-coefficient model to the two series and examine whether the estimated coefficients are occasionally compatible with bubble-like behaviour. After a brief specification search to determine the dynamic structure for our models, we settled for the following first-order autoregressive model

$$\Delta x_t = \alpha + \beta_t x_{t-1} + u_t; \qquad \beta_t = \bar{\beta} + \eta_t;$$

where α and $\bar{\beta}$ are constants, and $\{u_t\}$ and $\{\eta_t\}$ are independent i.i.d. processes with zero mean and variance σ_u^2 and σ_η^2, respectively. This model allows the coefficient β_t to vary randomly around the value $\bar{\beta}$.

Estimation and testing in the context of the random-coefficient autoregressive model given above can easily be carried out by using the Kalman filter to evaluate the likelihood function (assuming conditional Gaussianity) and to obtain smoothed estimates of β_t (see, e.g., Hamilton, 1994, Ch. 13). The maximum likelihood estimates of the model parameters for the prices and money supply series are reported in Table 2. The results clearly indicate that both series can be represented by a time-varying coefficient model with an autoregressive root that varies randomly around a value slightly larger than unity.

Figures 2 and 3 show the smoothed state series β_t (which represents the best estimate, in terms of mean squared error, of the state series based on all information available in the sample) together with ± 2 the root mean squared error around the corresponding smoothed estimate. It is evident from the plots that the period between 1923:06 and 1923:11 was characterized by explosive behaviour. The plots also show that prices suddenly stopped rising in December 1923, moving to a stationary regime (the event of stabilization followed the "monetary reform" on 15 October 1923; see Sargent, 1986). Crucially, however, the explosive episode is common to both the price and money series, implying that the most likely explanation for the 1922–1923 hyperinflation is the rapid growth in the money supply rather than the existence of a rational bubble.

4 Summary

This paper has examined the possibility of detecting the presence of explosive rational bubbles through an analysis of the stochastic properties of the relevant observable time series based on the class of autoregressive model with time-varying parameters. Our procedure involves testing for the presence of a random unit root in prices and underlying fundamentals and estimating random-coefficient models for the two series. Evidence of explosive behaviour in prices that is not accompanied by synchronous explosive behaviour in fundamentals would suggest the existence of a rational price bubble. The potential applicability of the proposed procedure has been illustrated through an analysis of German hyperinflation data.

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Table 1. $\hat{\eta}_T$ and $\hat{\eta}^*_T$ Tests[a]

	p							
	0	1	2	3	4	5	6	12
				Money				
$\hat{\eta}_T$	0.031	<u>1.277*</u>	1.006*	0.889*	0.788*	0.914*	0.831*	0.477*
$\hat{\eta}^*_T$	3.852*	<u>1.032*</u>	0.800*	0.707*	0.622*	0.729*	0.663*	0.395*
$\lvert\hat{\theta}\rvert$	0.919	0.440	0.431	0.418	0.417	0.421	0.415	0.411
				Prices				
$\hat{\eta}_T$	0.020	0.895*	<u>0.296*</u>	0.342*	0.178†	0.216*	0.136	0.061
$\hat{\eta}^*_T$	2.661*	0.900*	<u>0.479*</u>	0.645*	0.539*	0.676*	0.571*	0.231*
$\lvert\hat{\theta}\rvert$	0.882	0.007	0.363	0.452	0.561	0.587	0.626	0.649

[a]Underlined figures correspond to the value of p selected by the AIC. * and † indicate significance at the 0.05 and 0.10 levels, respectively.

Table 2. Maximum Likelihood Estimates[a]

	Money		Prices	
α	–0.618	(0.501)	–0.581	(0.349)
β	0.066	(0.018)	0.087	(0.016)
σ_u^2	0.026	(0.103)	0.202	(0.119)
σ_ε^2	0.001	(0.070)	0.001	(0.072)
Log-likelihood	–64.01		–76.00	

[a]Figures in parentheses are estimated standard errors.

Figure 1. Log of Wholesale Prices Index and Money

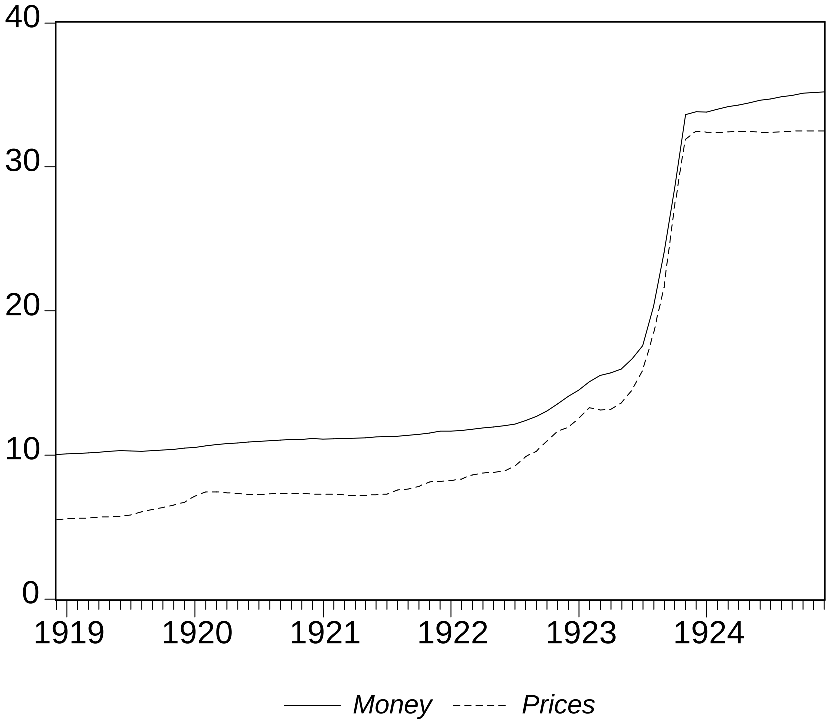

Figure 2. Smoothed Estimates Money Equation

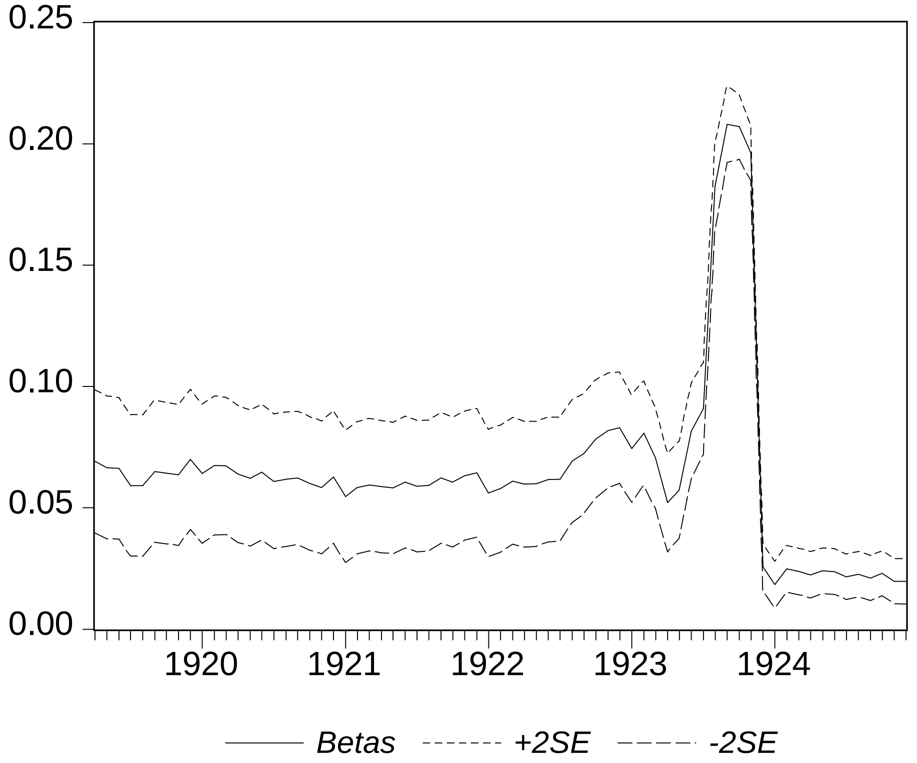

Figure 3. Smoothed Estimates Price Equation